Pseudorigidity method for solving the problem of limit equilibrium of rigid-plastic constructions

1.Design calculations , based upon the theory elasticity , cannot completely satisfy engineers and designers , because cannot answer to basic question about overload capability . Only design calculations of limit equilibrium of rigid-plastic constructions can answer to this question completely enough.

As a rule , such design calculations are made issue from hypothesis , that material of construction has rigid-plastic diagram Prandtl .This scheme of calculation gives qualitatively more correct results , then usual calculation , based upon law Hooke's , and allows more really estimate ultimate strength of construction due to different loads.

Universal algorithms for solving the problem of limit equilibrium have been created since the middle of the 60's.These algorithms are based upon two basic theorems about limit analysis - static and kinetics.

It was found , that with the help of above-mentioned theorems the problem of limit equilibrium can be formulated as a problem of linear programming (for linear yield) or nonlinear programming (for yield Guber-Mizes).

The method of linear programming conformably to calculation of rod systems got the most development in the reports Prager W. [1] and Chiras A. [2].

The method of linear programming conformably to plates and shells was widely used by Rganizin A.[3]. [3[contains more full bibliography about this problem.

Calculation of limit equilibrium with the help of linear and nonlinear programming has a few significant lacks:

- complexity and laboriousness preliminary preparation of problem for PC;
- necessity to use special program means , which are not in usual program packet for strength analysis.

Author worked out a new method about design calculation of limit equilibrium without above-mentioned lacks . The method is based upon analogy of relations between internal generalized efforts and generalized deformations in elastic system and between generalized efforts and velocities of change generalized deformations in rigid-plastic system.

Because later rigid-plastic deformation would be treated as an elastic deformation in the system with special constructed rigidities , this method could be called « pseudorigidity method ».

Basic idea of this method and a few test results are stated below.
2.Consider relations between moments (bending and twisting) and velocities of change curvature in rods with rectangular cross-section due to its rigid-plastic deform . It is known [4] , that using yield Guber-Mizes these relations become:

$$M_1 : M_2 = \dot{\chi}_1 : 1/3\dot{\chi}_2 , \quad (1)$$

where M_1 , M_2 - correspondingly bending moment and twisting moment;

$\dot{\chi}_1$, $\dot{\chi}_2$ - correspondingly velocities of change curvature and twisting in cross-section.

Moments M_1 , M_2 satisfy with equation:

$$M_1^2 + 3M_2^2 = M_s^2 , \quad (2)$$

where M_s - limit plastic bending moment in cross-section.

Consider arbitrary rod system . Suppose , that due to given type of external load rigid-plastic solution for this system is known . Bring in correspondence with this rod system another rod system with the same geometrical parameters , boundary conditions ,with the same load and with the following rigidities distribution:

-for nondeformed cross-sections ($M_1^2 + 3M_2^2 < M_s^2$)

$$EI_1 = GI_2 = \infty \ ; \quad (3a)$$

- for deformed cross-sections ($M_1^2 + 3M_2^2 = M_s^2$) rigidities are ultimate and $EI_1 : GI_2 = 1:1/3$ (3b)

In (3): I_1 - moment of inertia due to bending;

I_2 - moment of inertia due to twisting.

- for different deformed cross-sections ratio of rigidities is equal reverse ratio of velocities of change curvature.

Under ultimate load this elastic system will be have the same distribution of internal efforts and deformations as an initial rigid-plastic system. So , initial problem comes to seeking of distribution rigidities (3a),(3b) in elastic system.

For seeking this distribution of rigidities it was found sufficiently simple algorithm of iteration.

On each step of iteration process for every system's cross-section there are defined rigidities:

$EI_{1n}=K_nEI_{1,0}$; $GI_{2n}=K_nGI_{2,0}$ (4)

In (4) rigidities $EI_{1,0},GI_{2,0}$ satisfy with equation (3b);

K_n is defined by force parameters of solution , which was got on $(n-1)$ step and by values K_{n-1} .

Load P_n for calculation of the system on n step is defined on condition that yield will be achieved even if in one cross-section $(n-1)$ system and in all cross-sections inequality $M_1^2+3M_2^2\leq M_s^2$.

We can prove , that due to iteration process rigidities EI_{1n},GI_{2n} will be converged to distribution (3) , but load P_n - to solution of rigid-plastic problem. For different conditions of yield and for different combination force factors we can build the elastic system which will be equivalent to rigid-plastic system.

Using pseudorigidity method due to plates and shells scheme construction of iteration process remained the same , but pseudoheight h_n is variable parameter of rigidity.

3.Consider a few examples , which are solved by pseudorigidity method.

A. Solution the problem of limit equilibrium of circular two-hinged arch, loaded by concentrated force 2P (fig. 1).

Using pseudorigidity method arch was divided to 14 parts. Within the limits of each part parameter of rigidity doesn't change. The problem was solved by two variants: a) without taking into account of influence longitudinal force on overload capability of arch; b) with taking into account of influence longitudinal force. The problem was solved for different φ_0 .

Account influence longitudinal force was made for $K=\dfrac{M_s}{2RN_s}=0.01$

(M_S - limit moment in cross-section, N_s - limit longitudinal force).

Analysis showed, that pseudorigidity method determines parameters of limit equilibrium with accuracy 1% - 2% already for 9-10 iterations.

Dimensionless parameters $p=\dfrac{PR}{M_s}$ (P - half of limit load) for arch, received by theoretical method [4] and by pseudorigidity method, are compared in the tables 1 and 2 for 20th iteration.

B. Solution the problem of limit equilibrium of the rod, loaded by concentrated force in the middle of the span (fig. 2) .

Dimensionless parameters of limit force is: $\bar{p} = \frac{Pl}{M_s}$,

where P - limit load for rod, M_s - limit moment in cross-section . Due to 13th iteration - $\bar{p} = 2.97$; after 30th iteration - $\bar{p} = 3.0$ .Theoretical value in [5] - $\bar{p} = 3.0$.

Results of pseudorigidity method for another examples (see table 13.3 in [5]) also give accuracy ≈ 1% already to 10th - 15th iterations .

C. Basic purpose of pseudorigidity method is creation possibility of implementation rigid-plastic analysis in modern powerful program packets conformably to calculation of strength.

In order to show this possibility it was organized connection between block of transformation rigidities with external load and program packet for calculation of strength (« COSMOS»). Blocks of transformation parameters have form «exe.files» with capacity 31978 Kb for rods and 172908 Kb for plates. Also it was worked out command file SOLVE.BAT with capacity 1000 b. Calculations were executed with the help of above-mentioned program means and packet «COSMOS» for the rod (fig. 2).

This results also give $\bar{p} = 2.97$.

Summary .

Pseudorigidity method gives possibility considerably to expand region of strength analysis due to present packets «COSMOS» , « ANSYS « etc. with help of including to this region solving rigid-plastic problem.

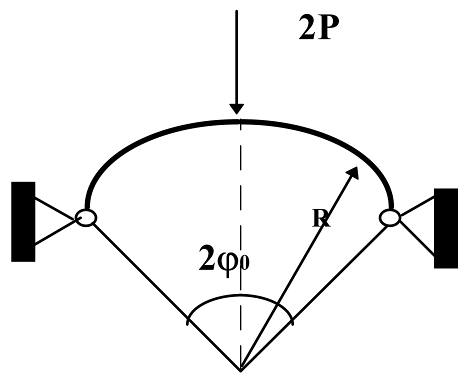

Fig. 1

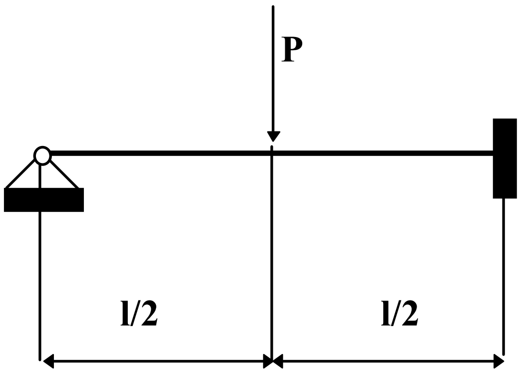

Fig. 2

Table 1. Results for arch, $N_s = \infty$, ($K = \dfrac{M_s}{2RN_s} = 0$)

φ_0	P (results of pseudorigidity method)	P (theoretical results)
20	19.4	19.5
30	12.9	12.95
40	9.6	9.6
50	7.6	7.6
60	6.3	6.3
70	5.3	5.3
80	4.6	4.6
90	4.0	4.0

Table 2. Results for arch ($K = \dfrac{M_s}{2RN_s} = 0.01$)

φ_0	P (results of pseudorigidity method)	P (theoretical results)
10	5.65	5.6
20	6.6	6.6
30	7.0	7.0
40	6.55	6.6
50	5.9	5.9
60	5.2	5.2

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